NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	LEVEL: 6
COURSE CODE: PBT 602S	COURSE NAME: PROBABILITY THEORY 2
SESSION: JANUARY 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr. D. NTIRAMPEBA
MODERATOR:	Dr. D. B. GEMECHU

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

ATTACHMENTS

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Excluding this front page)

Question 1 [25 marks]

1.1 Briefly explain the following terminologies as they are applied to probability theory.
(a) Boolean algebra $\mathcal{B}(S)$ [3]
(b) σ algebra [3]
(c) Measure on a $\mathcal{B}(S)$ algebra [3]
(d) Convolution of two integrable real-valued functions f and g [3]

1.2 Let $S = \{1, 2, 3\}$. Find:
(a) $\mathcal{P}(S)$ [2]
(b) size of $\mathcal{P}(S)$ [1]

1.3 Show that if m is a measure on $\mathcal{B}(S)$ and $c \geq 0$, then cm is a measure, where $(cm)(A) = c.m(A)$ [4]

1.4 Let X and Y denote the lengths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f(x, y) = \begin{cases} e^{-(x+y)} & ,x > 0, y > 0, \\ 0 & , \text{otherwise,} \end{cases}$$

then find the median value of Y. [6]

Question 2 [25 marks]

2.1 An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cumulative distribution function of X is

$$F(x) = \begin{cases} 0 & , \text{if } x < 1, \\ 0.4 & , \text{if } 1 \leq x < 3, \\ 0.6 & , \text{if } 3 \leq x < 5, \\ 0.8 & , \text{if } 5 \leq x < 7, \\ 1.0 & , \text{if } x \geq 7. \end{cases}$$

(a) Use $F(x)$ to compute $P(4 < X \leq 7)$. [2]
(b) What is the probability mass function of X? [5]
(b) What is the expected value of X? [3]

2.2 The joint probability density function of the random variables X, Y, and Z is

$$f(x, y, z) = \begin{cases} \frac{4xyz^2}{9} & , 0 < x < 1, 0 < y < 1, 0 < z < 3, \\ 0 & , \text{otherwise.} \end{cases}$$

Find the:
(a) joint marginal density function of Y and Z; [3]
(b) marginal density of Y; [3]
(c) $P(1/4 < X < 1/2, Y > 1/3, 1 < Z < 2)$. [3]

2.3 Given a random variable X, with standard deviation σ_X, and a random variable $Y = a+bX$, show that if $b < 0$, the correlation coefficient $\rho_{XY} = -1$, and if $b > 0$, $\rho_{XY} = 1$. [6]

Question 3 [20 marks]

3.1 Let X be a random with a probability density function $f(x)$ and a moment-generating function denoted by $m_X(t)$. Show that $m_X(t)$ packages all moments about the origin in a single expression. That is, $m_X(t) = \sum_{x=0}^{\infty} \frac{t^k}{k!} \acute{\mu}_k$. [5]

3.2 (a) Show that the characteristic function of a binomial variable X is $\phi_X(t) = E(e^{itX}) = (e^{it}p + (1-p))^n$. [5]
(b) Use this characteristic function to find the mean and variance of X. [5]

3.3 Show that the moment-generating function of random variable X, which takes value -1 and 1 with probability $\frac{1}{2}$, is $\frac{1+e^{2t}}{2e^t}$. [5]

Question 4 [30 marks]

4.1 Let X be a Poisson random variable with parameter λ. Derive the characteristic function of X and use it to find the mean of X. [8]

4.2 Let Y be continuous random variable with a probability density function $f(y) > 0$. Also, let $U = h(Y)$. Then show that

$$f_U(u) = f_Y(h^{-1}(u)) \frac{dh^{-1}}{du},$$

[7]

4.3 The random variables X and Y, representing the weights of creams and toffees, respectively, in 1 kilogram boxes of chocolates containing a mixture of creams, toffees, and cordials, have the joint density function

$$f(x,y) = \begin{cases} 24xy & ,0 \leq x \leq 1, 0 \leq y \leq 1,\ x+y \leq 1 \\ 0 & ,\text{elsewhere} \end{cases}$$

Find the joint probability density function of $Z_1 = X$ and $Z_2 = X + Y$. [10]

4.4 Let X and Y be independent Poisson random variables with parameters λ_1 and λ_2. Use the **convolution** formula to show that $X + Y$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$. [5]

END OF QUESTION PAPER